Mathematics Competition

Indiana University of Pennsylvania

2014

Do not turn this page until directed by the proctor to do so.

DIRECTIONS:

1. Please listen to the directions on how to complete the information needed on the answer sheet.
2. Indicate the most correct answer to each question on the answer sheet provided by blackening the `bubble' which corresponds to the answer that you wish to select. Make your mark in

Answer Key

1. D
2. D
3. E
4. D
5. A
6. B
7. A
8. B
9. D
10. C
11. E
12. B
13. C
14. C
15. C
16. D
17. B
18. C
19. C
20. E
21. D
22. E
23. A
24. A
25. E
26. E
27. A
28. B
29. D
30. D
31. E
32. C
33. D
34. D
35. D
36. D
37. B
38. B
39. B
40. B
41. A
42. D
43. B
44. A
45. C
46. D
47. A
48. C
49. C
50. C

1. Line segment AB has endpoints $(2; \quad 3)$ and $(\quad 4; 6)$. The coordinates of the midpoint of AB are:

   A. $(\quad 2; 3)$
   B. $(\quad 1; 3)$
   C. $\quad 3; \frac{9}{2}$
   D. $\qquad 1; \frac{3}{2}$
   E. None of these

---

2. The equation of the line perpendicular to $2x + 3y = 6$ and passing through the point $(8; 3)$ is:

   A. $2x + 3y = 25$
   B. $3x + 2y = 30$
   C. $2x \quad 3y = 7$
   D. $3x \quad 2y = 18$
   E. None of these

---

3. Let $a = \frac{3}{5}$, $b = \frac{1}{3}$, and $c = \frac{5}{2}$. Then, $ac^2 \quad bc + a$ is equal to:

   A. 109/15
   B. 121/60
   C. 7/2
   D. 13/4
   E. None of these

---

4. For triangle $4ABC$, you are given that $\cos(C) = 1{=}2$, the length of AC is 7, and the length of BC is 3. The length of side AB is:

   A. 10
   B. $\sqrt{69}$
   C. 4
   D. $\sqrt{37}$
   E. $\quad =6$

---

5. The graph which could be used to nd the solution to $y = \quad x + 2$ and $y = x^2$ is:

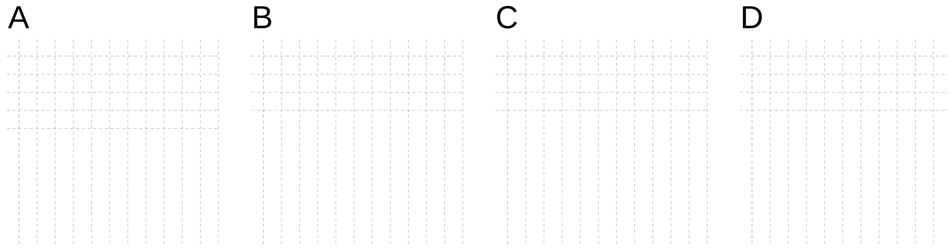
   A. Plot A.

   B. Plot B.

   C. Plot C.

   D. Plot D.

   E. None of these



---

6. The solution to the inequality $j2 \quad 4xj < 18$ is:

   A. $[\quad 5; 5]$

   B. $(\quad 4; 5)$

   C. $(\quad 1; \quad 4) \, [\, (5; 1)$

   D. $(\quad 1; \quad 4] \, [\, [5; 1)$

   E. $(\quad 1; 1)$

---

7. Let R denote the set of Real numbers, N denote the set of Natural numbers, and Q denote the set of Rational numbers. The true statement is:

   A. Every member of N is a member of Q

   B. Every member of R is a member of Q

   C. Every member of Q is a member of N

   D. Every member of R is a member of N

   E. All of the above statements are true

---

8. Let $f(x) = 4x + 5$, $g(x) = 3x \quad 1$, and $h(x) = 7 \quad 2x$. De ne $P(x) = (f \quad g)(x)$ and $Q(x) = (g \quad h)(x)$. Then, $(Q \quad P)(1 \quad z)$ is:

   A. $72z \quad 147$

   B. $72z \quad 58$

   C. $72z + 50$

   D. $72z + 168$

   E. None of these

---

9. Given the gure below with altitude BD, median BF and BE as the bisector of angle ABC, the valid conclusion is:

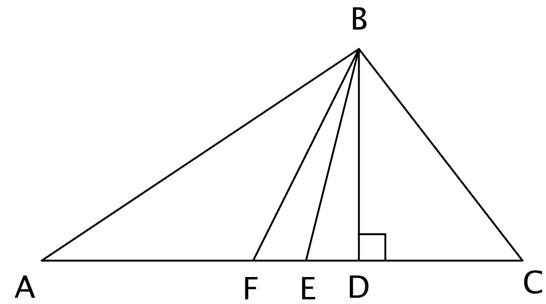
   A. $\backslash FAB = \backslash ABF$

   B. $\backslash ABF = \backslash CBD$

   C. $CD = EA$

   D. $CF = FA$

   E. None of these



10. Given that tan

13. Let $(x; y)$ be a solution to the system of equations $2x \quad 3y = 8$ and $4x + 3y = \quad 2$. Then, the product of x and y is equal to:

   A. 1

   B. 2

   C. 2

   D. 1

   E. 4

---

14. A farmer has 120 feet of fencing. He wants to put a fence around three sides of a rectangular plot of land, with the side of a barn forming the fourth side. The maximum area he can enclose is:

   A. 3600 square feet

   B. 30 square feet

   C. 1800 square feet

   D. 60 square feet

   E. None of these

---

15. An an-319(089-Tf 152 Tf -19.897 -19.289 Td [(E.)]TJet)]TJ0 g 0 G/F19 11.9552 Tf -19.897 -19.289 Tg72

17. Consider the system of equations $3u^3 \quad 3v = 6$ and $3u \quad v = 4$. A value of v in the ordered paired solutions $(u; v)$ is:

A. 12

B. 10

C. 0

D. 4

E. 10

---

18. The interval that contains all solutions to $2x(x \quad 2) = (x \quad 2)(x + 4)$ is:

A. $[\quad 6; 0]$

B. $[\quad 4; 3]$

C. $[0; 4]$

D. $[3; 6]$

E. $[5; 10]$

---

19. In the diagram, the circle has a radius of 5 and $CE = 2$. The length of BD is:

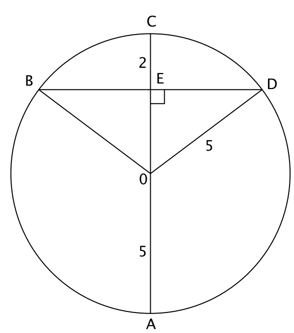
A. 12

B. 10

C. 8

D. 4

E. None of these



---

20. Suppose a and b are integers greater than 100 such that $a + b = 300$. A possible ratio of a to b is:

A. 9 to 1

B. 5 to 2

C. 5 to 3

D. 4 to 1

E. 3 to 2

21. The solution of $e^{2x} + e^{x} \quad 2 = 0$ is:

A. ln(2)

B. 1

C. ln(2)

D. 0

E. None of these

---

22. The vertex of the parabola $y = 3x^2 \quad 6x + 1$ is the point $(h; k)$ where $h + 2k$ is:

A.   4

B. 19

C. 5

D. 3

E.   3

---

23. The crescent moon M is bounded by the edges of two circles C_1 and C_2 with radii r_1 and r_2, respectively. Circle C_1 coincides with M along an angle $\ _1$ (measured in radians) while circle C_2 coincides with M for an angle $\ _2$ (in radians). The perimeter of M is:

A. $r_1 \ _1 + r_2 \ _2$

B. $r_1 \ _1 \quad r_2 \ _2$

C. $r_2 \ _1 \quad r_2 \ _2$

D. 2

25. The domain for the rational function $f(x) = \frac{x-5}{3x^3-13x^2-10x}$ is:

A. $(-\infty, \infty)$

B. $(-\infty, -2/3] \cup [-2/3, 0] \cup [0, \infty)$

C. $(-\infty, -2/3] \cup [-2/3, 0] \cup [0, 5] \cup [5, \infty)$

D. $(-\infty, -2/3) \cup (-2/3, 0) \cup (0, \infty)$

E. $(-\infty, -2/3) \cup (-2/3, 0) \cup (0, 5) \cup (5, \infty)$

---

26. The number of positive factors of 25200 that are not divisible by 10 is:

A. 18

B. 30

C. 47

D. 48

E. None of these

---

27. In a race, athletes run three laps around an oval track formed by a rectangle and two semicircles as shown. The length of a radius of each semicircle is 12 meters. The length of the top of the rectangle is equal to twice the diameter of the semicircle. The total distance the athletes run during the race is:

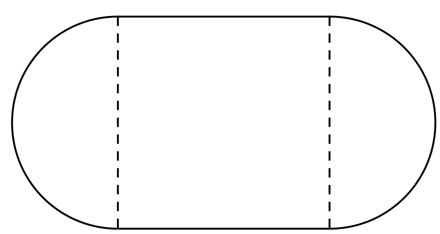
A. $288 + 72\pi$ meters

B. $96 + 24\pi$ meters

C. $288 + 144\pi$ meters

D. $144 + 72\pi$ meters

E. $96 + 12\pi$ meters



---

28. If $\log_2 5 = a$ and $\log_2 3 = b$, then $\log_2(0.9)$ in terms of a and b is:

A. $2a - b - 1$

B. $2b - a - 1$

C. $2b + a + 1$

D. $2a + b + 1$

E. None of these

---

29. The solution set of $\frac{2}{x+1} > \frac{1}{x\ \ 2}$ is:

A. $(\ \ 1;\ \ 1)\ [\ (2; 1)$

B. $(\ \ 1; 2)$

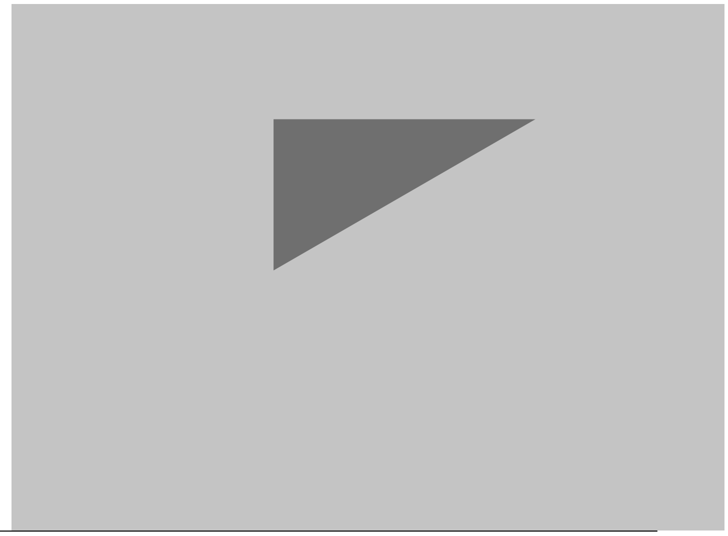
C. $(\ \ 1;\ \ 1)\ [\ (2; 5)$

D. $(\ \ 1; 2)\ [\ (5; 1)$

E. None of these

30.

37. Suppose that, in the gure below, the shaded right-angled triangle is such that the length of AB is $\sqrt{3}$ and the length of DB is 1. The area of the triangular region that lies outside of the circle is:



A. $=6$

B. $(3\sqrt{3} \quad)=6$

C. $(\quad 1)=2$

D. $\sqrt{3}=2$

E. None of these

38. Let n be the largest integer less than 10,000 that leaves a remainder of 1 when divided by any of the numbers 2, 3, 4, 5, 6, 7, or 8. The sum of the digits of n is:

A. 23

B. 16

C. 15

D. 13

E. 12

39. If $s = 1 + 3^1 + 3^2 + \quad + 3^8 + 3^9$, then the value of s is:

A. 3^{10}

B. $\frac{3^{10} \quad 1}{2}$

C. $\frac{3^9 \quad 1}{2}$

D. $\frac{1 \quad 3^8}{2}$

E. None of these

40. If $\sin(\ ) = \frac{\sqrt{3}}{2}$ and $\frac{\ }{2} < \quad < \quad$, then $\sin^{\ 1}(\cos(\ ))$ is:

A. 120

B. 30

C. 60

D. 60

E. 150

41. Let $\log_2(x) = 2\log_2(3) \quad \frac{3}{2}\log_2(9)$. Then $3x + 8$ is:

A. 9

B. 10

C. 11

D. 17

E. $\frac{25}{2}$

---

42. Carla and Joe are painting. It takes Carla 24 minutes to paint a wall alone, and it takes 28 minutes for Joe to paint the same wall by himself. They both start painting the wall together, but Joe has to quit to run an errand while Carla nishes. Carla continues to work for exactly the same amount of time that both she and Joe had already worked together. How long from the start of the job did it take to paint the wall?

A. $12\frac{2}{7}$ minutes

B. $13\frac{2}{3}$ minutes

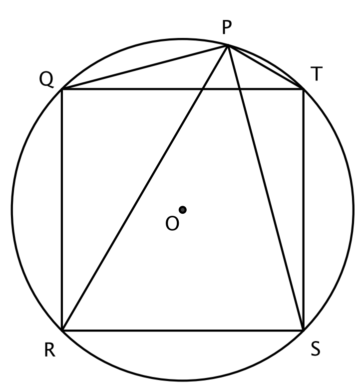
C. 14 minutes

D. $16\frac{4}{5}$ minutes

E. $17\frac{1}{2}$ minutes

---

43. In the circle shown with center O, the radius is 6. $QTSR$ is an inscribed square. De ne $w; x; y$, and z to be the lengths of segments PQ, PT, PR, and PS respectively. The value of $w^2 + x^2 + y^2 + z^2$ is:



A. 432

B. 288

C. 36

D. 144

E. None of these

---

44. For real numbers x and y, define $x \star y = x + y + xy$. Next, define a sequence of functions $\{f_1, f_2, f_3, \ldots\}$ recursively such that

$$f_1(x) = x$$

and for each natural number $n \geq 2$

$$f_n(x) = x \star f_{n-1}(x).$$

Then the coefficient of x^{10} in the function $g(x) = 1 + f_{25}(x)$ is:

A. $3{,}268{,}760$

B. $360{,}360$

C. 10

D. 250

E. None of these

---

45. An exact value for $\sin \frac{\pi}{16}$ is:

A. $\frac{1-\sqrt{2}}{2}$

B. q 2 / 2 + p

47. If $r = x = y$ and $s = (x \quad y) = (x + y)$, then $4r = (1 \quad r^2)$ is equivalent to:

A. $s \quad 1 = s$

B. $s + 1 = s$

C. $s = (s \quad 1)$

D. $s^2 \quad s$

E. $1 = (s + 1)$

---

48. The product of all the solutions to $\frac{\cos}{1 + \sin} + \frac{1 + \sin}{\cos} = 4$ on the interval $[\quad ; \quad]$ is:

A. $\frac{2}{18}$

B. $\frac{2}{36}$

C. $\frac{2}{9}$

D. $\frac{25 \quad ^2}{36}$

E. None of these

---

49. Two circles $C_1; C_2$ each with radius r are centered at the origin O and P, respectively. Circle C_1 is xed in the plane, but circle C_2 is rotating about C_1 in a counterclockwise manner while maintaining a point of tangency T. The x-coordinate of the point Q, which, before rotation through an angle , was the initial point of tangency is:

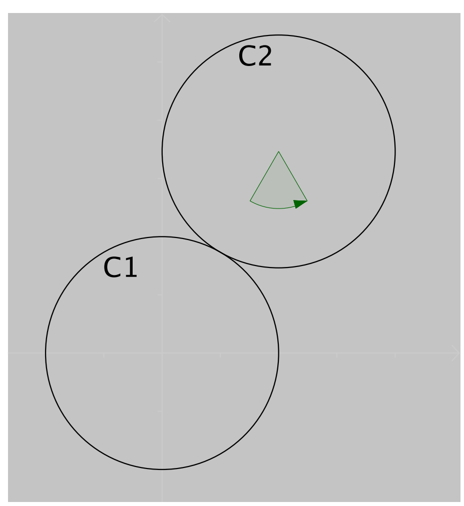
A. $2r \cos(2 \quad) + r \cos(\quad + 2 \quad)$

B. $2r \cos(\quad) + r \cos(\quad + \quad)$

C. $2r \cos(\quad) + r \cos(\quad + 2 \quad)$

D. $r \cos(\quad) + 2r \cos(\quad + 2 \quad)$

E. Impossible to determine



50. The multiplicative inverse of a 2×2 matrix A is the 2×2 matrix B such that $AB = BA = I_2$, where I_2 is the 2×2 identity matrix given by

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Suppose that A is a 2×2 matrix such that

$$7A^2 - 3A + 4I_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

The multiplicative inverse of A is:

A. Does not exist

B. $\frac{1}{7}A^2 - \frac{1}{3}A + \frac{1}{4}I_2$

C. $\frac{3}{4}I_2 - \frac{7}{4}A$

D. $\frac{7}{4}A^2 - \frac{3}{4}A$

E. None of these