Mathematics Competition

Indiana University of Pennsylvania

2015

DIRECTIONS:

1. Please listen to the directions on how to complete the information needed on the answer sheet.
2. Indicate the most correct answer to each question on the answer sheet provided by blackening the `bubble' which corresponds to the answer that you wish to select. Make your mark in such a way as to completely ll the space with a heavy black line. If you wish to change the answer, erase your rst mark completely since more than one response to a problem will be counted wrong. Make no stray marks on the answer sheet as they may count against you.
3. If you are unable to solve a problem, leave the corresponding answer space blank on the answer sheet. You may return to it if you have time.
4. Avoid wild guessing since you are penalized for incorrect answers. If, howev-32(er,)-524(y8rs.)-434(If,)-

Answer Key

1. A
2. B
3. E
4. C
5. D
6. A
7. D
8. B
9. A
10. E
11. E
12. D
13. E
14. B
15. C
16. A
17. E
18. E
19. B
20. B
21. D
22. C
23. B
24. C
25. A
26. D
27. C
28. B
29. D
30. C
31. C
32. B
33. C
34. D
35. E
36. B
37. D
38. C
39. D
40. B
41. B
42. B
43. D
44. A
45. A
46. A
47. C
48. C
49. A
50. E

1. The line with the greatest slope is:

   A. $9x \quad 7y = \quad 4$
   B. $12x \quad 17y = 1$
   C. $6x + 2y = 4$
   D. $2x \quad 3y = \quad 3$
   E. $y = x$

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2. A bag of rabbit food will feed 24 rabbits for 72 days. The number of days it will feed 16 rabbits is:

   A. 48
   B. 108
   C. 96
   D. 64
   E. 144

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3. Consider the following system of equations: $y = 13x \quad 1$ and $y = 10x + b$. When $(x;\ 1000)$ is a solution to the system, the value of b is:

   A. 0
   B. $3x \quad 1$
   C. 77
   D. $23x \quad 1$
   E. 230

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4. If $2^{x+1} = 50$, then:

   A. $x = 4$
   B. $5 < x < 6$
   C. $4 < x < 5$
   D. $x < 4$
   E. None of these

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5. Given that $\angle ABD = (9x - 53)$ and $\angle EBC = (7x - 27)$, then the values for $\angle ABD$ and $\angle EBC$ are:

   A. $\angle ABD = 13$ and $\angle EBC = 13$

   B. $\angle ABD = 128$ and $\angle EBC = 64$

   C. $\angle ABD = 13$ and $\angle EBC = 167$

   D. $\angle ABD = 64$ and $\angle EBC = 64$

   E. $\angle ABD = 83$ and $\angle EBC = 83$

9. A poll was taken, and it was found that 12 students went to the movies on Friday, 10 students went to the movies on Thursday, 4 went on both days, and 7 did not go on either day. The percentage of students who went to the movies only on Thursday is:

   A. 24%
   B. 31%
   C. 30%
   D. 8%
   E. 50%

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10. When x and y are nonzero, the expression $\dfrac{\frac{y^2}{x} \quad \frac{x^2}{y}}{\frac{1}{y} \quad \frac{1}{x}}$ is equal to:

   A. $x^2 + (xy)^2 + y^2$
   B. $x^2 + y^2$
   C. $x^2 + xy \quad y^2$
   D. $x^2 \quad xy + y^2$
   E. $x^2 \quad xy \quad y^2$

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11. The square of $(\log_{49} 7 \quad \log_8 64)$ is:

   A. 1
   B. $\frac{3}{2}$
   C. $\frac{3}{2}$
   D. $\frac{9}{4}$
   E. $\frac{9}{4}$

13. Suppose a circle is inscribed within a square with the radius equal to two. The circle touches the square at the midpoint of each side. Then, the combined area of the shaded regions is:

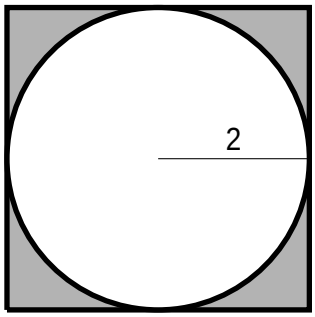
A. 16 + 4

B. 4 4

C. 16

D. 12

E. 16 4



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14. In a class there are 15 girls and 10 boys. Three students are randomly selected. The probability that 1 boy and 2 girls are selected is:

A. 626/118

B. 21/46

C. 3/25

D. 1/48

E. 25/56

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15. Of expressions A, B, C, and D given below, the expression that has a numerical value di erent from the others is:

A. $4 + 2\sqrt{3}$

B. $\dfrac{8 + \sqrt{48}}{2}$

C. $6 + \sqrt{48}$ $2 + \sqrt{12}$

D. $\sqrt{2}(\sqrt{8} + \sqrt{6})$

E. None of these, they are all equal

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16. Given that $f(x) = \log_2(x)$, $g(x) = 8^x$, and (f (

17. A Sportman's Club usually charges a $50 annual membership fee. Next year, the price will be increased to $60. Assume the Club can lose 4 members next year and still collect the same revenue as this year. Then the number of members in the club this year is:

A. 20 people

B. 26 people

C. 27 people

D. 28 people

E. None of these

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18. The area of a circle is 36 square units. The center of the circle is at (2; 3). The circle does not pass through the point:

A. (2; 3)

B. (4; 3)

C. (2; 9)

D. $(1; 3 + 3\sqrt{3})$

E. The circle passes through all of these points.

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19. The expression $x + 7$ divides evenly into the cubic polynomial $x^3 + ax^2$ $61x + 14$. The number of possible values of the variable a is:

A. 0

B. 1

C. 2

D. 3

E. Cannot be determined from the given information

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20. The solution set, in interval notation, for $\frac{(x \quad 1)^3(x \quad 3)}{x(x^2 \quad 11x + 24)}$ 0 is:

A. (1 ; 0) [[1; 8)

B. (0; 1] [(8; 1)

C. (0; 1] [[3; 8)

D. [1; 3) [(3; 8)

E. None of these

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21. The expression equivalent to $\sec(x)$ is:

A. $\cos(x) + \dfrac{\tan(x)}{\cos(x)}$

B. $\dfrac{\sin(x)}{\sec(x)} + \tan(x)$

C. $\tan(x) \quad \cos(x)\sin(x)$

D. $\tan(x)\sin(x) + \cos(x)$

E. $\dfrac{1}{\tan(x)\csc(x)} \quad \dfrac{\cos(x)}{\tan(x)}$

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22. Assume $x \neq 0$. If $x + y = 2xy$, then $\dfrac{y^2}{x^2}$ is equal to:

A. $4x^2 \quad 4x + 1$

B. $4x^2 + 4x + 1$

C. $4y^2 \quad 4y + 1$

D. $4y^2 + 4y + 1$

E. $4y^2 \quad 4xy + x^2$

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23. In a given sequence, the first term is 3 and each term after that is one less than three times the previous term. The sixth term of the sequence is:

A. 611

B. 608

C. 609

D. 607

E. 617

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24. The product of two negative real numbers is 25 and the sum of the two numbers is S. Then, S must satisfy:

A. $S = \quad 10$

B. $S \quad 15$

C. $S \quad 10$

D. $S \quad 10$

E. None of these

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25. The exact value of $\tan \frac{5}{8}$ is:

A. $\sqrt{2} - 1$

B. $\sqrt{2} + 1$

C. $\frac{\sqrt{2} - 1}{2}$

D. $\frac{\sqrt{2} + 1}{2}$

E. $\frac{\sqrt{2} + 1}{2}$

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26. The interval containing all solutions to $\sqrt{4x} + \sqrt{2x} = \sqrt{4x - 12} + \sqrt{2x + 12}$ is:

A. $(0; 2)$

B. $(1; 3)$

C. $(2; 5)$

D. $(4; 7)$

E. $(6; 10)$

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27. Let $ABCD$ be a quadrilateral with sides $AB = 6$, $BC = 5$, $CD = 3$, and $AD = 4$. Given that $\angle BAD$ and $\angle ADC$ are both right angles, the area of the quadrilateral is:

A. 24

B. 12

C. 18

D. Not possible to determine

E. None of these

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28. The number of liters of a 9% acid solution that must be mixed with 5 liters of a 3% acid solution and 4 liters of a 6% acid solution to obtain a solution that is 8% acid is:

A. 43

B. 33

C. 23

D. 13

E. 3

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29. If $10a \quad a^2 = 1$, the value of $a^2 + \frac{1}{a^2}$ is:

A. 20

B. $40\frac{1}{}$

33. The total combined area of the two figures below (not to scale) is:

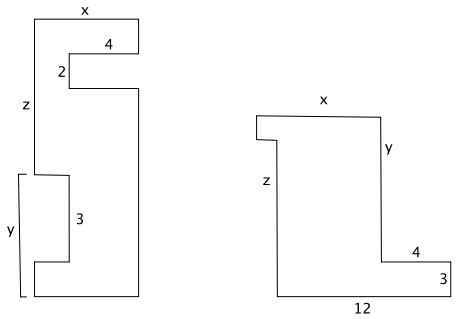
A. 320

B. 560

C. 2xy + 8z + 16

D. 2xz - 6x - 8z - 8

E. None of these



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34. If four experts and two trainees are working on a project, it takes one day to finish the project. If only two experts are working on a project, then it takes five trainees working with them to finish the project in one day. If one trainee was working alone on a project, the number of days it would take him or her to finish the project is:

A. 27=6 days

B. 5 days

C. 36=7 days

D. 8 days

E. 47=9 days

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35. A cube has a sur 0 1 -254.744 42.009 cm[]0 d 0 J 0.398 w 0 0 mie0 g 0 GBT/F15 11.9552 Tf 57.6 354.269 T

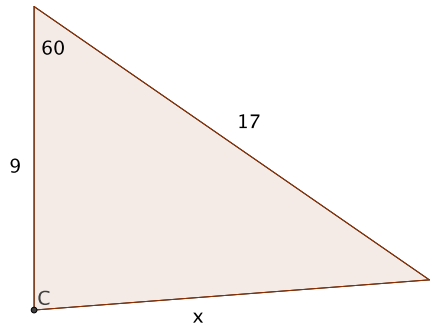
D.grkingC.

C. A6. days

B.35427g0G/F1811.9552Tf19.897119.428Td[E.)TJ0g0G165(are)811TfJ0.398w00m496.841770

37. In the triangle below (the figure may not be to scale), the value of x is:

A. $9\sqrt{3}$
B. $17\sqrt{3}$
C. $4\sqrt{13}$
D. $\sqrt{217}$
E. $\sqrt{370}$



38. A train travels to its destination and back during a trip. If it averages 80 miles per hour on the way and 90 miles per hour on the return, the train's average rate for the entire trip is:

A. 83 miles per hour
B. 85 miles per hour
C. $\frac{1440}{17}$ miles per hour
D. $\frac{720}{17}$ miles per hour
E. $\frac{1230}{17}$ miles per hour

39. You have a bag containing 6 red marbles, 4 blue marbles, and 8 purple marbles. In simplest form, the probability of pulling a red marble and not putting it back, then pulling a blue marble and putting it back, and then pulling a purple marble would be:

A. 16/459
B. 32/729
C. 8/153
D. 32/867
E. 3/52

40. If $\frac{t+3}{t-2} = x$ and $\frac{z+1}{z-2} = t$, then z is equal to:

A. $\frac{x+1}{x+4}$
B. $\frac{5x+5}{x+4}$
C. $\frac{x+3}{x-2}$
D. $\frac{x-4}{2x+2}$
E. None of these

41. A sector of a circle has a radius of 8 inches and an area of 8 square inches. The arc length of the sector is:

A. inches

B. 2 inches

C. 4 inches

D. 8 inches

E. None of these

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42. Two numbers are called a *twin prime pair* when each of the two values are prime and their di erence is 2. For example, 149 and 151 form a twin prime pair. The number of twin prime pairs that exist between 0 and 100 is:

A. 7

B. 8

C. 9

D. 10

E. None of these

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43. For a positive real number a, consider

$$2\log_b(x) = 2\log_b(1 \quad a) + 2\log_b(1 + a) \quad \log_b \quad \frac{1}{a} \quad a \quad ^2 :$$

Then x is equal to:

A. a

B. a

C. a^2

D. a

E. b

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44. The sum of the distinct real solutions to $x^3 + 21x^2 \quad x = x^4 + 20$ is:

A. 1

B. 1

C. 3

D. 11

E. 11

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45. In the right triangle below, find the values of x, y, and z:

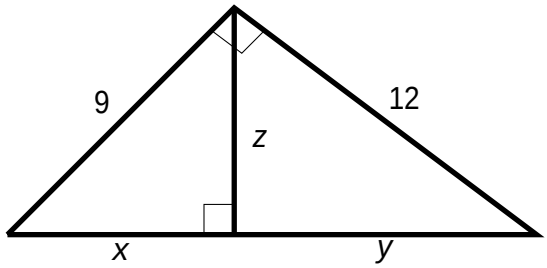
A. $x = \frac{27}{5}, y = \frac{48}{5}, z = \frac{36}{5}$

B. $x = 27, y = 48, z = 36$

C. $x = \frac{15}{2}, y = \frac{15}{2}, z = \frac{3\sqrt{39}}{2}$

D. $x = \frac{15}{2}, y = \frac{15}{2}, z = \frac{3\sqrt{11}}{2}$

E. None of these



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46. Consider the following equation:

$$\frac{14x + 10}{x^4 + 3x^3 \quad 3x^2 \quad 7x + 6} = \frac{A}{x + 2} + \frac{B}{x \quad 1} + \frac{C}{(x \quad 1)^2} + \frac{D}{x + 3}$$

Then $A^2 + B^2 + C^2 + D^2$ is equal to:

A. 12

B. 16

C. 20

D. 25

E. 36

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47. In a cafeteria, there is a circular table seating 10 students. Two seating arrangements are said to be distinct if there exist two students who are neighbors in one arrangement but not in the other arrangement. The number of distinct seating arrangements is:

A. 10!

B. $\frac{10 \quad 9}{2}$

C. $\frac{9!}{2}$

D. 9!

E. None of these

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48. An interval containing the solution to $\sqrt[3]{x} + \sqrt{3x + 32} = 6$ is:

A. (-3,-1)

B. (-1,0)

C. (0,1)

D. (1,2)

E. (2, *1*)

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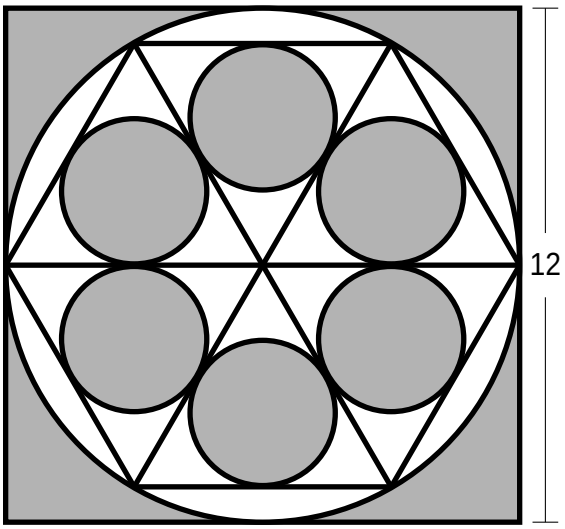
49. A circle is inscribed within a square with a side of 12 units. A regular hexagon is then inscribed within the circle. The hexagon is then divided into six equilateral triangles, and nally, a circle in inscribed within each of these triangles (see the diagram below). The area of the shaded region is:



A. 18(8) square units

B. 18 square units

C. 144 $\frac{63}{2}$ square units

D. 36(4) square units

E. 36 square units

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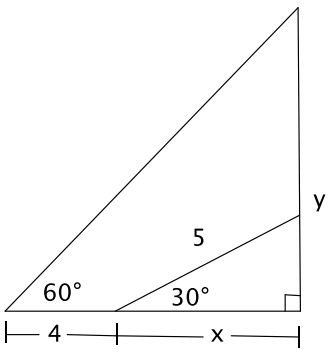
50. Given the following diagram (not to scale) the exact area of the larger triangle is:



A. $\frac{1}{2}$ $89\sqrt{3} + 240$ square units

B. $\frac{1}{4}$ $139\sqrt{3} + 240$ square units

C. $\frac{1}{8}$ $89\sqrt{3} + 160$ square units

D. $\frac{1}{8}$ $139\sqrt{3} + 260$ square units

E. None of these
