1. A perfect numberis a positive integer that is equal to the sum of all of its divisors (other than itself). The smallest perfect number is 6, which is the sum of 1, 2, and 3. Another example of a perfect number is:

   A. 16
   B. 42
   C. 28
   D. 8
   E. None of these

---

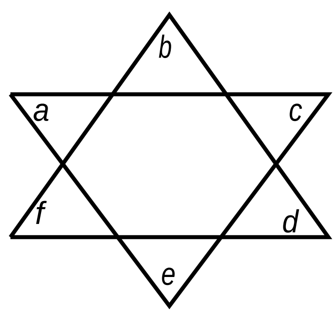
2. If $7x - 5y = 43$ and $3x + 2y = 6$, then $x - y$ is equal to:

   A. -7
   B. 1
   C. 31
   D. 55
   E. None of these

---

3. In the figure below, the value of $a + b + c + d + e + f$ is:

   A. 270
   B. 120
   C. 540
   D. 360
   E. 240



---

4. Find the sum of the first six terms in the geometric sequence $2, -6, 18, -54, \ldots$

   A. -364
   B. 364
   C. -468
   D. 468
   E. None of these

---

5. A farmer used 220 feet of fencing to enclose a rectangular garden. If the area of the field is 2800 square feet, then the length of the longer side of the garden is:

A. 50 feet

B. 60 feet

C. 70 feet

D. 80 feet

E. 100 feet

---

6. The complex fraction $\dfrac{x^{-1} + y^{-1}}{\frac{3}{xy}}$ simplifies to:

A. $\dfrac{2xy}{3x + 3y}$

B. $\dfrac{x^2y - xy^2}{3}$

C. $\dfrac{xy}{3x + 3y}$

D. $\dfrac{xy}{x^2y + xy^2 + 3}$

E. $\dfrac{x + y}{3}$

---

7. The value $\sin^2(56^\circ) + \sin^2(34^\circ)$ is equal to:

A. 0

B. $1 + \cos^2(34^\circ)$

C. 1

D. $1 + \sin^2(34^\circ)$

E. $\tan^2(56^\circ)$

---

8. The expression $\log_5 \sqrt[3]{\dfrac{25}{a^2 + b^2}}$ is equal to:

A. $\frac{1}{3}[2 - \log_5(a^2 + b^2)]$

B. $\frac{1}{3}\log_5(25) - \log_5(a^2 + b^2)$

C. $3[2 - \log_5(a^2 + b^2)]$

D. $3\log_5(25) - \log_5(a^2 + b^2)$

E. $\frac{1}{3}[2 - \log_5(a^2) + \log_5(b^2)]$

9.

13. If the sum of five consecutive positive integers is A, then the sum of the next five consecutive integers in terms of A is:

A. $A + 1$

B. $A + 5$

C. $A + 25$

D. $2A$

E. $5A$

---

14. Recall the definition of a perfect number from question #1. We may also define a semiperfect number to be a positive integer that is equal to the sum of some of its divisors (other than itself). An example of a semiperfect number is 12, which is the sum of 2, 4 and 6. We did not use divisors 1 and 3, so 12 is not perfect. Another example of a semiperfect number is:

A. 10

B. 16

C. 32

D. 56

E. None of these

---

15. The value of $\sin(75^\circ)$ is equal to:

A. $\frac{1-\sqrt{2}}{\sqrt{3}}$

B. $\frac{1+\sqrt{3}}{\sqrt{8}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1+\sqrt{3}}{\sqrt{2}}$

E. $\frac{1-\sqrt{3}}{\sqrt{2}}$

---

16. $AB + CD = AAA$, where AB and CD are two-digit numbers and AAA is a three-digit number. A, B, C, and D are distinct positive integers. The value of C is:

A. 1

B. 3

C. 7

D. 9

E. None of these

---

17. The inner square is constructed using the midpoints of the outer square. Let P be the midpoint of $\overline{AB}$. The angle is:

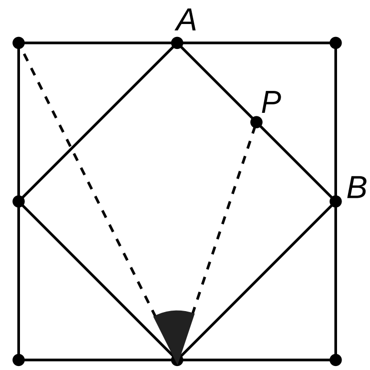
A. 22:5

B. 30

C. 45

D. 60

E. None of these



---

18. One solution of $\frac{33}{2x^2 + x \quad 6} \quad \frac{x \quad 2}{3x + 6} = 2$ is $x = \quad 3$. The other solution is:

A. $\frac{43}{14}$

B. $\frac{47}{14}$

C. $\frac{53}{26}$

D. $\frac{73}{14}$

E. $\frac{165}{26}$

---

19. Let $m > 1$ be a positive integer and write $k = \log_{\frac{1}{m}}(y_k)$. Then, the sum $\sum_{i=1}^{1} y_i$ is equal to:

A. m

B. $\frac{1}{m}$

C. $m \quad 1$

D. $\frac{1}{m \quad 1}$

E. None of these

20. The intersection of two sets A and B, denoted by $A \cap B$, consists of all objects that are in both sets. In set-builder notation,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

For each natural number n, define an interval of real numbers according to the equation,

$$A_n = \left(-3 - \frac{1}{n}, 7 + \frac{1}{n}\right).$$

Let $X = A_1 \cap A_2 \cap A_3 \cap \cdots$. In other words, X consists of all real numbers that are in each of the sets A_n for $n = 1, 2, \ldots$. The set X can also be expressed as:

A. $(-3, 7)$

B. $(-3, 7]$

C. $[-3, 7)$

D. $\emptyset$

E. None of these

---

21. The cost of gas rose by 2 cents per gallon from last week to this week. Last week Steve bought 20 gallons of gas at the old price. This week he bought 10 gallons at the new price. Altogether, Steve spent \$66.20 on gas. The old price for one gallon of gas is:

A. \$1.52

B. \$1.54

C. \$2.18

D. \$2.20

E. \$2.22

---

22. The sum of all solutions of $(x^2 + 6x + 2)^2 - 3(x^2 + 6x + 2) = 54$ is:

A. -12

B. 3

C. 16

D. -42

E. None of these

---

23. The option that contains all solutions to $e^{2x} - 5e^x - 14 = 0$ is:

A. $x = \ln 7$

B. $x = -2$ and $x = 7$

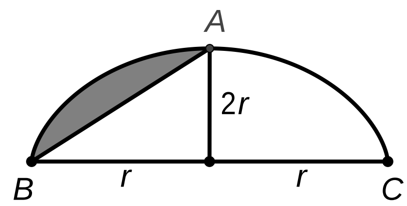
C. $x = 7$

D. $x = \ln(-2)$ and $x = \ln 7$

E. None of these

---

24. The total area under the cycloid BAC and above the base is 3 r^2. The area of the shaded region is:



A. $\frac{1}{2}$ r^2

B. r^2

C. $\frac{3}{4}$ r^2

D. $\sqrt{2}$ r^2

E. None of these

---

25. A function p whose domain and range is the set of real numbers R is called a projection provided that for every $x_1; x_2$ in R and for all constants $a; b$ in R,

$$p(ax_1 + bx_2) = ap(x_1) + bp(x_2) \quad \text{and}$$
$$p(p(x_1)) = p(x_1):$$

Suppose that x_0 is a real number in the domain of p. The value of $p(x_0 \quad p(x_0))$ is always equal to:

A. x_0

B. $p(x_0)$

C. $x_0 \quad p(x_0)$

D. 1

E. None of these

---

26. The expression $\sin^2(2 \ ) + (2\cos^2 \quad 1) \ 1 \quad 2\sin^2$ may be simpli ed to:

A. 1

B.

C. =2

D. 2

E. None of these

---

27. The solution set of $5^{\frac{x^3+4x^2+3x}{x+3}} = 15625$ is:

A. f 3g

B. f 2g

C. f2g

D. f 3; 2g

E. None of these

---

28. A right triangle is shown below. The value of $x + z$ is:

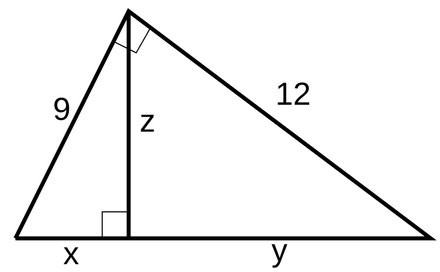
A. 9

B. 9.15

C. 10.4

D. 12.6

E. 15



29. A poll was taken, and it was found that 12 students went to the movies on Friday, 10 students went to the movies on Thursday, 4 went on both days, and 7 did not go on either day. The percent of all students polled who went to the movies only on Thursday is:

A. 40%

B. 6%

C. 10%

D. 24%

E. 30%

30. The second smallest number in the list 25^{60}, 16^{30}, 32^{24}, 27^{40}, 4^{120} is:

A. 25^{60}

B. 16^{30}

C. 32^{24}

D. 27^{40}

E. 4^{120}

31. When 4 is added to two numbers, the ratio is 5 : 6. When 4 is subtracted from the same two numbers, the ratio is 1 : 2. If the two numbers are x and y and $x < y$, then the value of $2x + y$ is:

A. 20

B. 22

C. 24

D. 26

E. 28

32. At what point or points, if any, does the graph of the given function cross its horizontal asymptote?

$$f(x) = \frac{3x^2 + 4x + 1}{x^2 + 3} :$$

A. (3; 2)

B. (2; 3)

C. (0; 1)

D. The function does not have a horizontal asymptote

E. The graph of the function does not cross the horizontal asymptote

---

33. The number 730400 has 2 trailing zeroes. Also, the symbol ! means factorial, or the product of an integer and all the positive integers below it (e.g. 4! = 4 3 2 1 = 24). The number of trailing zeros in the number 2019! is:

A. 195

B. 380

C. 403

D. 502

E. 614

---

34. A commercial airplane is detected 1000 miles north, as it is ying north with a speed of 700 miles per hour. Thirty minutes later, an interceptor plane ying with a speed of 800 miles per hour is deployed. The amount of time it will take the interceptor plane to reach the other plane is:

A. 12.5 hours

B. 13.0 hours

C. 13.5 hours

D. 14.0 hours

E. 14.5 hours

---

35. The area of the given shape is:

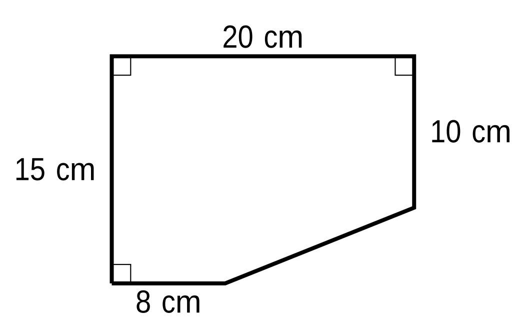
A. 240 cm^2

B. 250 cm^2

C. 260 cm^2

D. 270 cm^2

E. 280 cm^2



36. For a natural number k, a binary string of length k is an expression of the form

$$B = b_1 b_2 \cdots b_k,$$

where either $b_i = 0$ or $b_i = 1$ for $i = 1, 2, \ldots, k$. The weight of a binary string B, denoted by $w(B)$, is defined according to the equation

$$w(B) = b_1 + b_2 + \cdots + b_k.$$

The binary string B is even provided that $w(B)$ is an even integer and the binary string B is odd provided that $w(B)$ is an odd integer. The number of binary strings of length k having even weight is equal to:

A. 2^k

B. $2^{(1/2)k}$

C. 2^{k-1}

D. $\frac{1}{2}k$

E. None of these

---

37. The vertices of the inscribed square bisect the sides of the second square. The ratio of the area of the outer square to the area of the inscribed square is:

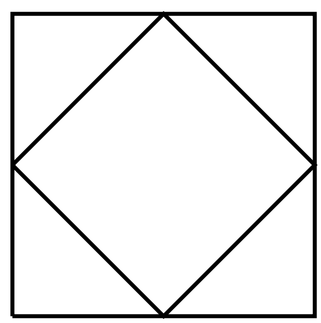
A. 2 : 1

B. 1 : 2

C. $\sqrt{2} : 1$

D. $1 : \sqrt{2}$

E. 4 : 1



---

38. Assume the area of $\triangle PQR$ is 1, and further assume that the ratio of lengths is $|PT|/|PQ| = k$. The area of the larger triangle is

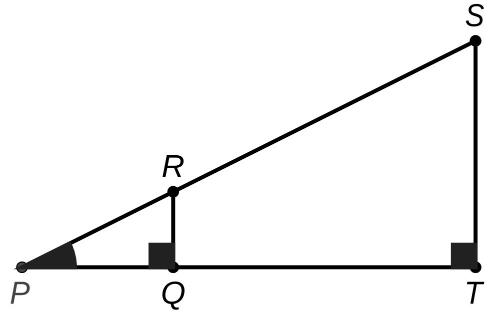
A. $\frac{1}{2}k$

B. k

C. $\frac{1}{2}k^2$

D. $2k^2$

E. None of these



---

39. The number of terms in the sequence 3, 6, 12, 24, . . . which are between the values 1000 and 100,000 is:

A. 3

B. 7

C. 9

D. 12

E. None of these

---

40. The sum of all   that are solutions to $3 \; \sin^2 \quad \cos^2 \quad = 13 \sin \quad 8$ where   is:

A. 2

B.

C. 0

D. =3

E. None of these

---

41. If a polynomial $P(x)$ with integer coe cients is divided by $x^2 + x \quad 6$, the remainder is $121x \quad 199$. If the same polynomial $P(x)$ is divided by $x^2 \quad x \quad 2$, the remainder is $15x + c$ where c is some integer. The remainder when $P(x)$ is divided by $x^2 + 4x + 3$ is:

A. $136x \quad 186$

B. $280x + 278$

C. $106x \quad 214$

D. $327x + 147$

E. $98x \quad 156$

---

42. Pierre Fermat, a French mathematician who lived from 1601-1665, theorized that every prime number of the form $4n + 1$, for $n \quad 1$, is the sum of two squares in one and only one way. For example, 13 can be written in the form $4(3) + 1$ and is equal to $9 + 4$. Now, let A be the largest prime number less than 100 that can be written in the form $4n + 1$. Then, we write $A = p^2 + q^2$. The value of j$p^2 \quad q^2$j is:

A. 39

B. 77

C. 13

D. 11

E. 65

---

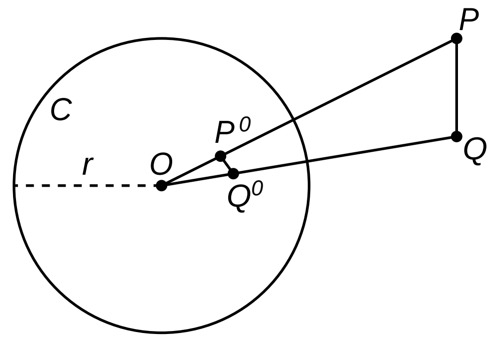
43. If $jx + 3j \quad a = b \quad j1 \quad xj$ has a solution, then the statement which must be true is:

A. $a + b = 0$

B. $a + b \quad 4$

C. $a + b <$

47. The inversion across the circle C with radius r is a map de ned as follows: for any point R (excluding the origin), (R) is the point on the ray $\overrightarrow{OR}$ with a distance $r^2/|OR|$ from the origin. A related process, called stereographic projection, has been used to make maps. Here, we have the inverses $P' = (P)$ and $Q' = (Q)$. The area $A(\ OP'Q')$ is:



A. $\frac{r^2}{|OQ|^2|OP|^2}A(\ OQP)$

B. $\frac{r^4}{|OQ|^2|OP|^2}A(\ OQP)$

C. $\frac{r^4}{|OQ||OP|}A(\ OQP)$

D. $\frac{r^2}{|OQ||OP|}A(\ OQP)$

E. None of these

---

48. For a xed natural number $n > 1$, modular arithmetic performed on two integers pertains to remainders upon division by n. In particular, for integers a and b,

$$a \ {}_n\, b = \text{the remainder when a+b is divided by n and}$$
$$a \ {}_n\, b = \text{the remainder when ab is divided by n.}$$

For example, if $n = 8$, then $3 \ \ 6 = 1$ since $3 + 6 = 9$ and the remainder when 9 is divided by 8 is equal to 1. Similarly, $2 \ \ 7 = 6$ since $2 \ 7 = 14$ and the remainder when 14 is divided by 8 is equal to 6.

If X is a subset of integers, then the ${}_n$ multiplicative identity for X is an element e of X for which

$$a \ {}_n\, e = a$$

for all elements a in X. It is possible that a given subset X does not have such an identity element, in which case, the ${}_n$ multiplicative identity does not exist.

For $n = 10$ and $X = \{2; 4; 6; 8\}$, the ${}_{10}$ multiplicative identity for X is equal to:

A. 2

B. 4

C. 6

D. 8

E. None of these

49. Given that $\log_3(p) = \log_4(q) = \log_{\frac{16}{3}}(p + q$

Answer Key

1. C
2. E
3. D
4. A
5. C
6. E
7. C
8. A
9. B
10. C
11. E
12. D
13. C
14. D
15. B
16. D
17. C
18. A
19. D
20. E
21. D
22. A
23. A
24. A
25. E
26. A
27. C
28. D
29. D
30. D
31. A
32. B
33. D
34. C
35. D
36. C
37. A
38. E
39. B
40. B
41. B
42. E
43. B
44. D
45. D
46. A
47. B
48. C
49. D
50. D