1. In class last week you talked about integration by substitution, where you use a change of variables to make an integration problem easier.

   <u>Example</u> Use substitution to calculate the value of the definite integral

2. Another topic involving integration is the idea of the of a function over an interval . The average value

Example

Find the average value of the upper part of the ellipse on the interval ,
and find the points on the ellipse where that average value is achieved.

3. Another topic covered in class was recovering quantities from their rate of change. Given the rate of change of a quantity , We know the following:

   A. The in between and is

   B. Given the initial value , the future value of at time is

This last equation can be interpreted as: future value = initial value + net change.

Example

Use the Fundamental Theorem of Calculus to find the position and velocity at time of an object moving along a straight line with the following characteristics:

2. If you assume that , the coordinates of the Sun (at one focus point) would be either as shown in the picture, or , which may be easier to use for calculations. Now, let denote the square of the distance from the planet to the Sun. Integrate over the interval to find the average value of this distance, and find the coordinates of the planet when it is exactly this distance from the Sun.

At Earth's surface, the acceleration due to gravity is approximately (with local variations). However, the acceleration decreases with distance from the surface according to Newton's Law of Gravitation; at a distance of meters from Earth's surface, the acceleration is actually given by

where is the radius of the Earth.
For this problem, we suppose that a projectile is launched upward with an inh39where

2. Integrate both sides of this equation with respect to , using the fact that when ,
, to show that

3. Use the fact that at the maximum height of the projectile to determine that the maximum height is

and show that the value of needed to put the projectile into orbit (the escape velocity) is